PHY489 Week 4:
Internal Symmetries

Flavour Symmetries:

- Proton and neutron have very similar masses, so we may define a new basis of "nucleons"

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad ; \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

similar to the spin-$\frac{1}{2}$ formalism.

Define isospin I and the third component I_3 (as for J_3).

These are components in some abstract coordinate space.

These components have nothing to do with real spin or rotations.

Ultimately, just an interpretation of quark number conservation.

$$p = \left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle, \quad n = \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

"isospin doublet"

Heisenberg: Strong interactions are invariant under rotations in "isospin space".

Noether: isospin conserved in strong interactions.

Pions: isospin $I=1$ (three charge states)

$$\pi^+ = |1,1\rangle \qquad \pi^0 = |1,0\rangle, \qquad \pi^- = |1,-1\rangle.$$

Isospin at **quark level:** $u = |\tfrac{1}{2}, \tfrac{1}{2}\rangle$ $\quad d = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$ $\quad s = |0,0\rangle$

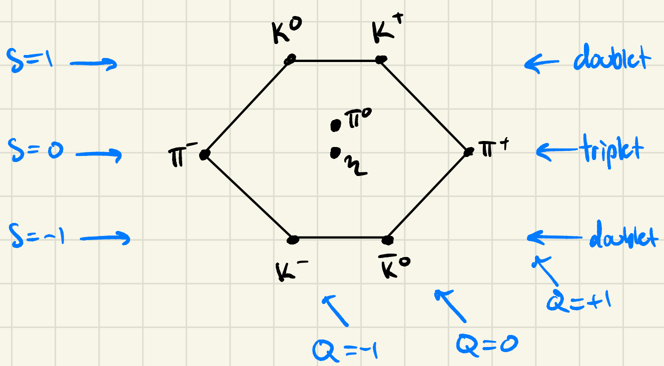
Other flavours are also isospin singlets (c, b, t)

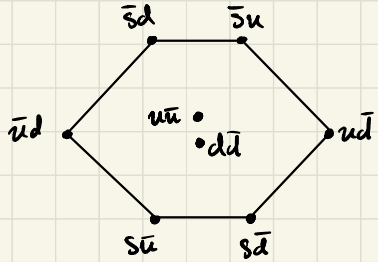
Mesons: containing only u, d quarks can have only isospin 1 or 0.

$\pi^-, \pi^0, \pi^+ \rightarrow$ isospin triplet

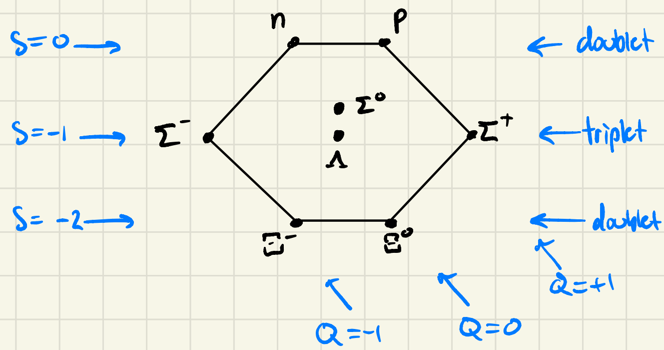
$\eta \rightarrow$ isospin singlet.

Octet:





Baryon Octet:



Parity:

$P(\vec{r}) = -\vec{r}$ (eigenvalues ± 1)

→ Any polar vectors $\vec{r} = (x, y, z)$

→ Scalars invariant $P(a) = a$.

→ Axial (pseudo) vectors invariant $P(\vec{L}) = P(\vec{A} \times \vec{B}) = \vec{L}$.

→ Pseudo scalars $q = \vec{a} \cdot (\vec{b} \times \vec{c})$

Parity conservation: all fundamental particles have intrinsic parity.

QFT: Fermion parity is opposite to that of its antiparticle

Boson parity is the same as its antiparticle.

→ Assign positive parity to quarks (negative for antiquarks)

→ Parity of a composite system is given by the product of the parity of its constituents, with an addition angular momentum contribution of $(-1)^{\ell}$ (ℓ = orbital angular momentum).

↳ Mesons $(-1)^{\ell+1}$

↳ Baryons $(-1)^3 (-1)^{\ell+1} = (-1)^{\ell}$

↳ Individual Hadrons : just a particle property.

Parity violation: (Colbalt) beta-emission is preferentially opposite the direction of nuclear spin, violating parity conservation

$$Co^{60} \rightarrow Ni^{60} + e^- + \bar{\nu}_e .$$

ie. $n \rightarrow p + e^- + \bar{\nu}_e$

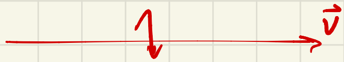
Parity is not conserved in weak decays, since the spin axial vector should not change direction under parity.

Helicity: As the choice of z-axis for spin measurement, use the axis defined by the particle's velocity:

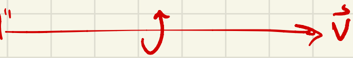
[units Helicity] = $\frac{M_s}{s}$.

A particle of spin-$\frac{1}{2}$ (fermion) can therefore have helicity ± 1:

+1: "right handed"

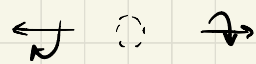


−1: "left handed"



It is not Lorentz invariant unless the particle is massless. (One can always "flip" helicity of a massive particle by always transforming into another frame with particle velocity $v' > v$)

Pion decay: $\pi^0 \rightarrow \mu^- \bar{\nu}_\mu$

$\bar{\nu}_\mu$  μ^-

Helicity 0

Spins must be anti-aligned
∴ Must have same helicity.

Experimentally, only right-handed (anti) neutrinos are observed (by measuring μ^- helicity)

Parity violation in weak decays:

- All neutrinos are left-handed
- All antineutrinos are right-handed.

Parity operations on ν_L should give ν_R - which we know does not exist.

Charge Conjugation: inverts all internal quantum numbers while leaving energy, mass, momentum, and spin unchanged.

This includes inverting charge, lepton number, baryon number, and strangeness.

Taking a particle into its antiparticle

$$C|n\rangle = |\bar{n}\rangle.$$

strong interactions, E+M is invariant!

Thus most particles are not eigenstates of C, except for photons and all entries in meson octet.

Eigenvalues are ± 1: $C|x\rangle \rightarrow \pm|x\rangle = |\bar{x}\rangle$.

Charge conjugation is not a symmetry of the weak interaction.

$$C|\nu_L\rangle \to |\bar{\nu}_L\rangle \quad \boxed{\text{NO}}$$

$$CP|\nu_L\rangle \to |\bar{\nu}_R\rangle \quad \boxed{\text{YES}}$$

In strong interactions, can extend C by combining it with an isospin transformation:

Rotations of π about I_2 takes $I_3 \xrightarrow{R_2} -I_3$. Hence

$R_2\,\pi^+ \to \pi^-$, so

$C R_2\,\pi^+ \to \pi^+$.

Any meson composed of only u and d quarks / antiquarks is an eigenstate of the "G-parity" operator $G \equiv CR_2$.

$C \equiv (-1)^{\ell+s}$.

so $G(\pi) = (-1)^1(1) = -1$.

CP Violation : SM contains only left and right-handed neutrinos, antineutrinos.

In weak processes:

$C|\nu_L\rangle = |\bar{\nu}_L\rangle$ $\boxed{\text{No}}$ → violation.

$P|\nu_L\rangle = |\nu_R\rangle$ $\boxed{\text{No}}$ → violation

$CP|\nu_L\rangle = |\bar{\nu}_R\rangle$ $\boxed{\text{No}}$ → flavour mixing neutrinos not allowed!

Quark Flavour Mixing:

Strongest Coupling: up/down-type quarks of same generation.

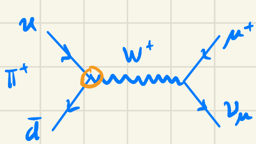
Next Strongest: —— " —— of neighbouring generations

Weakest: —— " —— of 1st, 3rd generations.

ie.

*Weak decays of s quarks require su coupling (generational transition)

Cabibbo Angle: Explains relative branching ratios for weak decays into leptons:



Need "form factors" to explain how this takes place in a bound state.

$u\bar{s}W^+$ coupling stronger than $u\bar{d}W^+$.

Define the original quark basis (of each generation):

$$u_L^i = (u_L, c_L, t_L)$$

$$d_L^i = (d_L, s_L, b_L)$$

Then let $u_L^{i'}$, $d_L^{i'}$ be the basis which diagonalizes the Higgs coupling (2). The two are related by the unitary transformations

$$u_L^i = \mathcal{U}_u^{ij} u_L^{j\,\prime}$$

$$d_L^i = \mathcal{U}_d^{ij} d_L^{j\,\prime}$$

Then transforming into a basis where these quarks are allowed to intergenerationally "mix" via the $W^\pm$ boson coupling: ie,

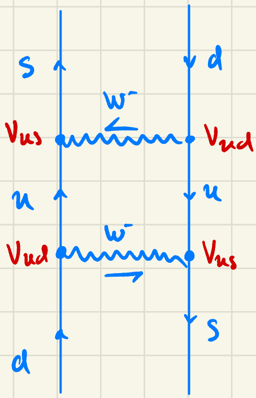
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$\uparrow$

Cabibbo - Kobayashi - Maskawa (CKM) matrix for quark coupling. 3 rotations and a phase.

CKM matrix describes the probability of a weak-coupling transistion interaction from one flavour of quark j to another quark i (V_{ij}).

Example: observed CP violation in the K^0 system:



Neutral Kaons are pseudoscalars $\left[P = (-1)^{\ell+1}\right]$

so:

$$P|K^0\rangle = -|K^0\rangle \qquad\qquad CP|K^0\rangle = -|\bar{K}^0\rangle$$

$$C|K^0\rangle = |\bar{K}^0\rangle \quad \rightarrow \quad CP|\bar{K}^0\rangle = -|K^0\rangle$$

So via diagonalization, we have the normalized eigenstates

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad CP\,|K_1\rangle = |K_1\rangle$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad CP\,|K_2\rangle = -|K_2\rangle.$$

But this isn't experimentally true -

→ What really occurs is that the weak interaction acts on "long-lived" and "short-lived" neutral kaon states which are <u>not</u> CP eigenstates:

$$|K_L^0\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$$

Thus CP is not a good symmetry of the weak interaction.

Experimental Determination of CKM matrix:

$$V_{ij} = \begin{pmatrix} 0.97373 \pm 0.00031 & 0.2243 \pm 0.0008 & 0.00382 \pm 0.00020 \\ 0.221 \pm 0.004 & 0.975 \pm 0.006 & 0.0408 \pm 0.0014 \\ 0.0086 \pm 0.0002 & 0.0415 \pm 0.0009 & 1.014 \pm 0.029 \end{pmatrix}$$

Essentially:

Consider a unitary chiral transformation between quarks relating quark 'mass eigenstates' to 'weak eigenstates'

$$u_R^i \to W_u^{ij} u_R^j \qquad u_L^i \to U_u^{ij} u_L^j$$

$$d_R^i \to W_d^{ij} d_R^j \qquad d_L^i \to U_d^{ij} d_L^j$$

These transformations leave the vector fields $J^{\mu 0}$ (Z^0 boson) and $J^{\mu \gamma}$ (photon) invariant, but $J^{\mu \pm}$ ($W^\pm$ bosons) are not invariant! Consider $J^{\mu +}$:

$$J^{\mu +} = \frac{1}{\sqrt{2}} \bar{u}_L^i \gamma^\mu d_L^i \longrightarrow \frac{1}{\sqrt{2}} \bar{u}_L^i \gamma^\mu \underbrace{(U_u^\dagger U_d)^{ij}}_{V^{ij} \text{ CKM matrix}} d_L^j$$

Let $\mathcal{L}_m = -\lambda_d^{ij} \overline{Q}_L^i \cdot \phi\, d_R^j - \lambda_u^{ij} \epsilon^{ab} \overline{Q}_{La}^i \phi_b^\dagger u_R^j + h.c$

be the most general renormalizable gauge-invariant coupling Lagrangian, with $\lambda_{d,u}^{ij}$ being general (not necessarily symmetric or Hermitian) complex-valued matrices.

→ CP transformation interchanges $\mathcal{L}_m$ operators without changing coefficients: $\lambda_{d,u}^{ij} \to (\lambda_{d,u}^{ij})^*$.

Eliminate T-dependence via chiral rotation:

$$\lambda_u \lambda_u^\dagger = U_u D_u^2 U_u^\dagger \qquad \text{(Left-handed chirality)}$$

$$\lambda_u^\dagger \lambda = W_u D_u^2 W_u^\dagger \qquad \text{(Right-handed chirality)}$$

which implies

$$\lambda_u = U_u D_u W_u^\dagger$$ where we take the positive square root of the D_u^2 eigenvalues.

and D_u^2, D_u is diagonal (obviously).

The quark masses are then related by

$$m_u^i = \frac{1}{\sqrt{2}} D_u^{ii} v$$

$$m_d^i = \frac{1}{\sqrt{2}} D_d^{ii} v$$

v: vacuum expectation value of Higgs field

$v = \langle 0 | \phi | 0 \rangle$.